

Uniaxial and Biaxial Substrate Effects on  
Uniform Finlines and Finline Step Discontinuities

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**Abstract**

Substrate materials exhibit inherent anisotropy which affects the dispersive characteristics of uniform finlines and finline discontinuities with increasing frequency. An accurate model is presented which accounts for uniaxial or biaxial anisotropy. The error involved in neglecting such substrate properties is discussed. In addition, the effect of anisotropy on the step discontinuity is analyzed and a comparison of the accuracy of the transverse resonance method and the mode matching technique is provided.

**Introduction**

The uniform finline properties on isotropic substrates have been analyzed by several authors who adopted a variety of techniques [1] - [7]. In addition the finline step discontinuity has been characterized by mode matching methods [8], [9], the TLM method [10] and the transverse resonance technique [11]. It was assumed in all of these models that the substrate material under consideration was isotropic. Recently it was shown that many substrate materials in use exhibit anisotropy which may be inherent to the material or acquired during the material manufacturing process [12]. It was also demonstrated in [12] that in modelling various structures pertinent to integrated circuits for microwave and millimeter wave applications, neglecting substrate anisotropy leads to modeling errors which may become very serious with increasing frequency. One of the aims of this article is to investigate the effects of uniaxial and biaxial substrate anisotropy on the dispersive properties of the uniform finline and on the finline step discontinuity. In addition a thorough comparison of the transverse resonance and mode matching methods will be presented.

**Analytical Formulation**

Although the methods adopted here can be applied to other structures including unilateral finline, antipodal finline or shielded microstripline and suspended substrate lines, here the problem is formulated for the bilateral finline. The cross section of the structure under consideration is shown in Figure 1a. The substrate used is in general biaxial with a permittivity tensor assumed to be of the form

$$\hat{\epsilon} = \epsilon_0 \begin{bmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix}$$

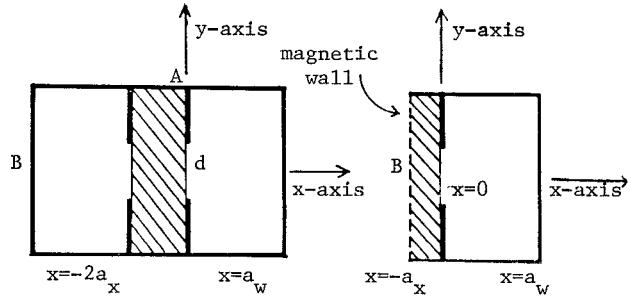


Figure 1.  
Cross section of the fin-line in waveguide

where  $\epsilon_0$  is the free space permittivity. The substrate is centered in the waveguide and the printed fin thickness is negligible. With the permittivity in tensor form, the wave equations in the substrate region can be derived from Maxwell's equations. In general the wave equations are coupled partial differential equations given by

$$\frac{\epsilon_{yy}}{\epsilon_{xx}} \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} - \left(1 - \frac{\epsilon_{zz}}{\epsilon_{xx}}\right) \frac{\partial^2 E_z}{\partial y \partial z} + k_o^2 \epsilon_{yy} E_y = 0 \quad (1a)$$

$$\frac{\epsilon_{zz}}{\epsilon_{xx}} \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} + \frac{\partial^2 E_z}{\partial y^2} - \left(1 - \frac{\epsilon_{yy}}{\epsilon_{xx}}\right) \frac{\partial^2 E_y}{\partial y \partial z} + k_o^2 \epsilon_{zz} E_z = 0 \quad (1b)$$

In general, the finline in the waveguide is excited by the dominant  $TE_{10}$  mode of the empty waveguide. Therefore, because of symmetry, a magnetic wall can be put at  $x = -a_w$ , and only the half-structure shown in Figure 1b need be considered. All field quantities are Fourier transformed via

$$\begin{cases} \bar{E}(n, x) \\ \bar{H}(n, x) \end{cases} = \int_{-b/2}^{b/2} \begin{cases} \bar{E}(x, y) \\ \bar{H}(x, y) \end{cases} e^{jk_n y} dy \quad (2)$$

where

$$k_n = \begin{cases} 2n\pi/b, n = 0, 1, 2, \dots \text{for } E_z \text{ odd mode} \\ (2n-1)\pi/b, n = 1, 2, 3, \dots \text{for } E_z \text{ even mode} \end{cases}$$

Decoupling of Equations 1a and 1b yields e.g. for  $E_z$

$$\frac{d^4}{dx^4} E_z(x, n) + (e_1 + g_1) \frac{d^2}{dx^2} E_z(x, n) + (e_1 g_1 - e_2 g_2) E_z(x, n) = 0$$

where  $e_1, e_2, g_1, g_2$  are constants dependent on the components of  $\bar{\epsilon}$ ,  $k_n$ ,  $k_o$  and the propagation constant  $\beta$ .

The boundary value problem is completed by solving for the dispersive properties of the propagation constant  $\beta$ . This is achieved by adopting the Galerkin procedure with expansion and testing functions which satisfy the edge condition at the slot edges. Also the power-voltage definition is used to calculate the effects of anisotropy on the finline characteristic impedance.

The effect of anisotropic substrates on the finline step discontinuity shown in Figure 2 is also considered. The transverse resonance method is applied to this end [11] as well as the mode matching technique [9]. A comparison between the two methods and their accuracy will be discussed in general and particular results will be presented for anisotropic materials.

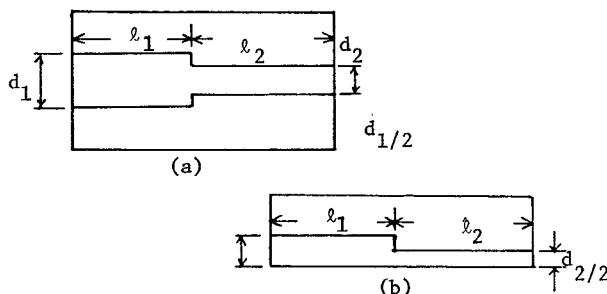


Figure 2  
Finline Step Discontinuity

#### Numerical Results

##### (a) Uniform Finline

Numerical values of the effective dielectric constant i.e.  $\epsilon_{eff} = (\beta/k_o)^2$  and characteristic impedance have been obtained. As a first example, the anisotropic effects on the finline characteristics are theoretically investigated for  $\epsilon_r = 3.0$  by changing dielectric constants in the x, y, z directions to 3.5 respectively. The  $\epsilon_{eff}$  computa-

tions show that for  $\epsilon_{xx} = 3.5$  the difference is about 0.7% for  $\epsilon_{yy} = 3.5$  about 7% and for  $\epsilon_{zz} = 3.5$  less than 0.1% for WR-28, over the frequency range of only the dominant mode propagation. Due to the fact that the substrate is thin, most of the power is propagated in the air region and this makes the characteristic impedance insensitive to the anisotropic effect. For single mode frequency range the finline has very much the same characteristics as the ridged waveguide. The characteristic impedance is sensitive to the change of propagation constant near the cut-off frequency by observing the equation

$$Z_{oc} = \frac{Z_{o\infty}}{\sqrt{1 - \left(\frac{\lambda}{\lambda_o}\right)^2}}$$

This explains the reason that at lower frequencies the anisotropy has stronger effects on the characteristic impedance. Comparison of  $Z_{oc}$  for the parameters of Figure 3 is shown in Figure 4. The characteristics of finline for PTFE both  $\epsilon_{xx} = 2.45$ ,  $\epsilon_{yy} = 2.89$ ,  $\epsilon_{zz} = 2.95$  and  $\epsilon_r = 2.45$  and a variety of other useful substrates will also be discussed.

##### (b) Finline Step Discontinuity

Numerical results of the scattering parameters of a single step discontinuity are obtained by both the transverse resonance and mode matching methods. The substrate is PTFE-cloth  $\epsilon_{xx} = 2.45$ ,  $\epsilon_{yy} = 2.89$ ,  $\epsilon_{zz} = 2.95$ . The comparisons are shown in Figure 5. Because in mode matching method the power flow is directly related to the finline discontinuity, it is observed that at about the same computer cost the results by the mode matching method are more stable and have better convergent behavior than those by the transverse resonance method.

#### Conclusions

The effect of anisotropy on the characteristics of the uniform finline and the finline with a step discontinuity is demonstrated. It is shown that neglecting anisotropy results in erroneous models of the finline propagation constant, characteristic impedance and step discontinuity equivalent circuit. In addition, a thorough comparison of the accuracy of the transverse resonance method and the mode matching technique is provided in the characterization of finline step discontinuities.

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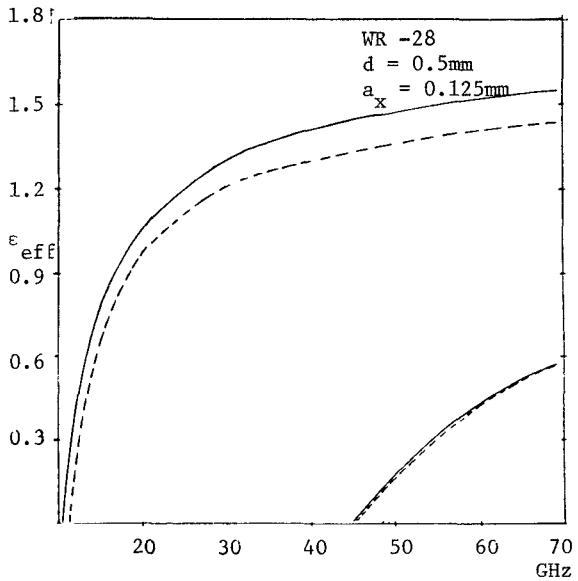


Figure 3  
Dispersion characteristics of fin-line,  $d = 0.5\text{mm}$

(a)  $\epsilon_r = 3.0$  -----  
 (b)  $\epsilon_{xx} = \epsilon_{zz} = 3.0, \epsilon_{yy} = 3.5$  —————

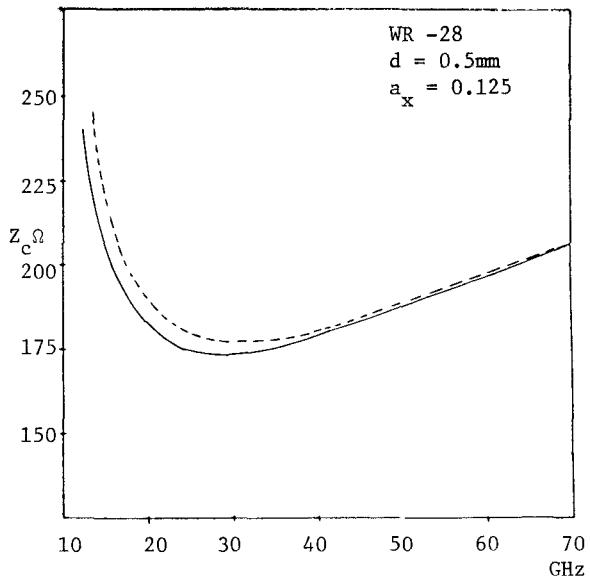


Figure 4  
Characteristic impedance of fin-line,  $d = 0.5\text{mm}$

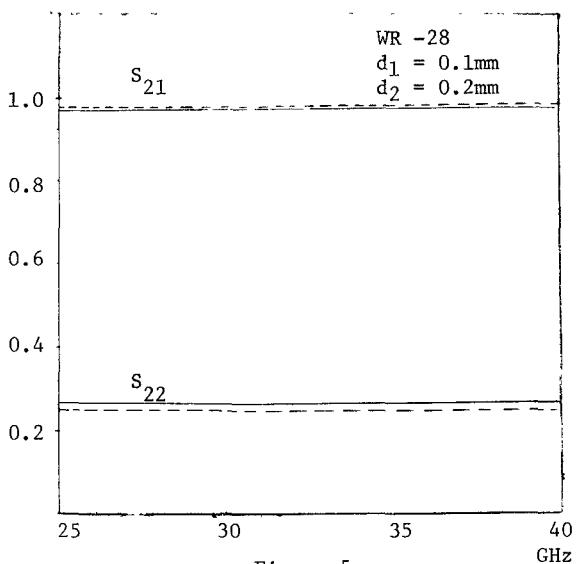


Figure 5  
Magnitude of the scattering parameters

$$\epsilon_{xx} = 2.45, \epsilon_{yy} = 2.89, \epsilon_{zz} = 2.95$$

— by mode matching method

— by transverse resonance method